Lecture 29

Monday, March 22, 2021 2:53 PM

* Prayer
* Spiritual thought
* Answering questions....

Vector fields

$f = x^3 + xy^2 + y^2 \rightsquigarrow$ potential function

$F = \langle 3x^2 + y^2, 2xy + 2y \rangle \rightsquigarrow$ conservative vector field.

$f_x = 3x^2 + y^2 \rightsquigarrow f(x,y) = x^3 + xy^2 + C(y)$

$f_y = 2xy + C'(y)$

$\rightsquigarrow C'(y) = 2y$

$\rightsquigarrow C(y) = y^2$

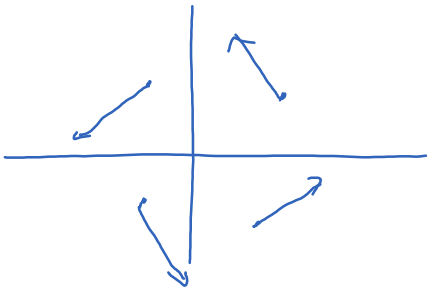
Ex (non-conservative vector field)

$F(x,y) = \langle y^2, x^2 \rangle$

$y^2 = f_x \rightsquigarrow f = xy^2 + C(y)$

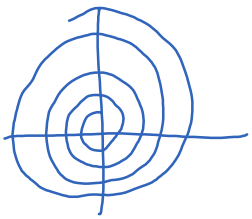
$f_y = 2xy + C'(y) = x^2$

$\rightsquigarrow$ there is no such f!



$f(x,y) = \langle y, x \rangle$

Examples of vector fields: gravity, velocity, magnetic field, gradient vectors, force, ...



$r = f(\theta) = e^{\theta} \sin\theta$

$\theta \to \infty \; : r \to \infty$

$\theta \to -\infty \; : r \to 0$

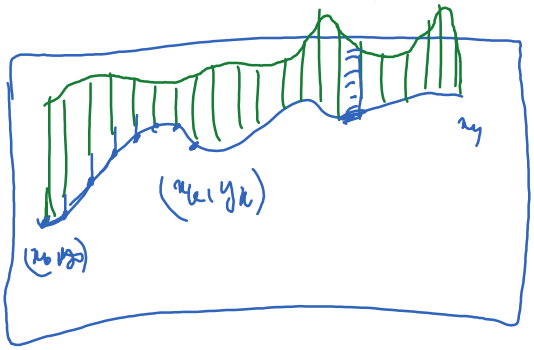
Plot vector fields on Mathematica:

VectorPlot [{x^2, xy}, {x, 0, 1}, {y, 0, 1}]

Line integral

integral
- "single" integral: $\int_a^b$, $\int_{[a,b]}$
- double integral: $\iint_D$
- triple integral: $\iiint_E$
- iterated integral: $\int_a^b \int_c^d \cdots$

Can we integrate a function $f(x,y)$ over a curve instead of a region?



$$s(t) = \int_a^t |r'(\tau)|\, d\tau$$

$$ds = s'\,dt = |r'(t)|\,dt$$

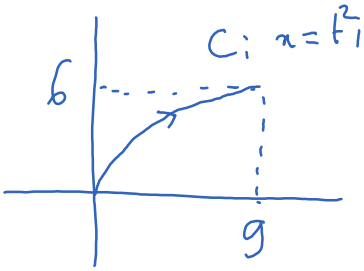
$$\sum f(x_k, y_k)\, |r'(t_k)|\, \Delta t$$

$$\approx \int_a^b f(x(t), y(t))\, \underbrace{|r'(t)|\, dt}_{ds}$$

$$= \text{“}\int_C f(x,y)\, ds\text{”}$$

integral of a scalar function over a curve.

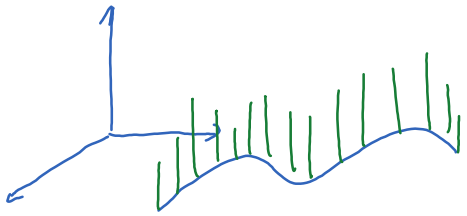
Ex



$\int_C y\,ds = ?$

Mathematica:

How to plot function $f(x,y)$ on top of a curve C?



$C: (x(t), y(t), 0)$

Surface: $(x(t), y(t), s)$

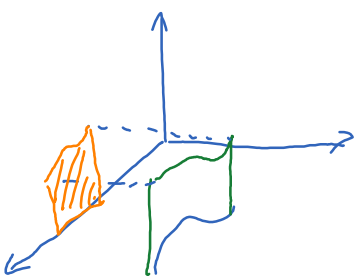
$t \in [a,b]$, $s \in [0, f(x(t), y(t))]$

ParametricPlot3D[{t², 2t, s}, {t, 0, 3}, {s, 0, 2}]

$\int_C f(x,y)\,ds$ is the area of the wall.

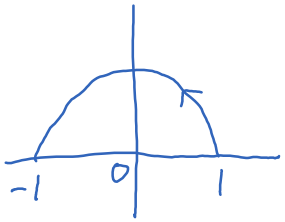
or mass of a wire

* Line integral wrt x or y



$\int_C f(x,y)\,dx$ = area of the shadow of the fence.

Ex $f(x,y) = x+y$



$$\int_C (x+y)\,dx = \int_0^{\pi} (\cos t + \sin t)(-\sin t)\,dt$$

$$= \cdots\cdots$$